Condition Assessment and Monitoring of Deteriorating Concrete Structures with Bayesian Methods

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ABSTRACT

One of the major factors affecting structural performance in time is deterioration of its components due to environmental conditions. Because large uncertainties are associated with the process of structural deterioration, probabilistic inference methods are more suitable for strength assessment and prediction. This contribution proposes a new framework of condition monitoring and remaining strength prediction for deteriorating concrete structures. A Bayesian dynamic linear model is used to describe the dynamics of the condition monitoring parameters. This model incorporates a certain deterioration model and will be used to describe the process of structural performance deterioration. When monitoring information becomes available, the evolution trend could be predicted and updated within the Bayesian framework. Meanwhile, the Cumulative Bayes Factors (CBF) are calculated to detect abnormalities in the parameter sequences. The advantage of using cumulative Bayes factors is that it can track changes of the sequence structure (evolution trend) even if outlier data at certain time points exist. Hence, it can avoid false alarms and can be used in consecutive monitoring, giving a timely warning when an out-of-range deterioration/accident occurs and providing information for optimal maintenance strategies.

INTRODUCTION

Civil structures, especially bridges are continuously subjected to attack from the surrounding environment. In contrast to severe damages that caused by vehicle collisions or earthquakes, environmental damage and structural degrading occurs gradually over time, and often goes undetected until significant damage has occurred. For reinforced concrete bridges, environmental attack causes minor to significant damages, including cracks and corrosion of embedded steel reinforcement. Corrosion is often caused by the presence of chloride ions, particularly in case of de-icing salts and in marine environments (Enright and Frangopol 1998, Val et al. 1998). The effects of some factors on corrosion have been analyzed (Vu and Stewart 2000), and several prediction models have been developed (Almusallam et al. 1996). Corrosion decreases the cross-sectional area and the strength of reinforcement, ultimately reducing the service life of RC structures. Structural strength degradation due to corrosion is a time-dependent process. An accurate prediction of strength degradation is important to evaluate the service condition of RC bridges. In the following parts, a new framework for condition prediction and monitoring of deteriorating concrete structures is proposed to assess the structural performance at present as well as a time horizon in the future. The Bayesian dynamic linear model is used to describe the dynamics of the structural parameters. It incorporates a certain deterioration model and will be used to describe the process of structural strength deterioration. Moreover, the cumulative Bayes factors are incorporated to detect abnormalities in the parameter sequences, giving a timely warning when an out-of-range deterioration/accident occurs and providing information for optimal maintenance strategies.

BAYESIAN DYNAMIC LINEAR MODEL

Although it is usually not possible to measure the state of a structure directly, it could be evaluated by using observations of its related physical parameters such as strains, deflections, etc. from monitoring processes or load tests. While the main area in which Dynamic Linear Models (DLM) (West and Harrison 1997) are used is modelling observations collected over time for purposes of forecasting or detecting structural states, it is suitable for structural condition assessment and performance prediction. The main purpose of this part is to explore the Bayesian analysis of DLM.

Dynamic Linear Model and Bayesian prediction

The model used by DLM is actually a sequence of models which are updated at each time step, justifying the expression ’dynamic’. The characteristic of interest or unknown states of a system in the time series is modelled as θ which is a vector of parameters. The main goal is to analyse the evolution of θ, on which the forecasting is based as well, since the prediction also depends on how θ behaves over time. The DLM consists of two basic equations: a state equation and an observation equation.

Observation equation: $$y_t = F_t \cdot \theta_t + v_t \qquad v_t \sim N(0, V_t) \tag{1}$$

State equation: $$\theta_t = G_t \cdot \theta_{t-1} + w_t \qquad w_t \sim N(0, W_t) \tag{2}$$

where F_t is the design matrix of y_t, G_t the evolution matrix of θ_t, v_t the measurement error vector and w_t the evolution error vector.

The details of using the DLM equations for the recursive one-step forward forecast of the (posterior) distribution in the Bayesian framework are summarized as follows.

Step 1: (initialization): Given the initial information set, determine the distribution of the initial variations as $(\theta_0|I_0) \sim N(m_0, C_0)$ with the estimated mean m_0 and variance C_0. Set $t = 0$ and choose estimated value for V_0 (or calculate sequentially in case unknown).

Step 2: (prior estimation): The posterior distribution of θ_t at current time interval t is employed to estimate the prior distribution of θ_{t+1} at time $t+1$, $(\theta_{t+1}|I_t) \sim N(a_{t+1}, R_{t+1})$, where $a_{t+1} = G_{t+1} m_t$, $R_{t+1} = G_{t+1} C_t G'_{t+1}/\delta$, and $W_{t+1} = G_{t+1} C_t G'_{t+1}(1-\delta)/\delta$. δ is a discount factor, which is used to structure evolution error matrices and lies between 0 and 1.

Step 3: (forecast): The mean and variance of the prior distribution of θ_{t+1} are used to predict the posterior distribution of $(y_{t+1}|I_t) \sim N(f_{t+1}, Q_{t+1})$ at time $t+1$: $f_{t+1} = F'_{t+1} a_{t+1}$, $Q_{t+1} = G'_{t+1} C_t G_{t+1} + V_{t+1}$.

Step 4: (update): As time moves forward, once the newly observed data at time $t+1$ (i.e., y_{t+1}) becomes available, update the posterior distribution as $(\theta_{t+1}|I_{t+1}) \sim N(m_{t+1}, C_{t+1})$, where $m_{t+1} = a_{t+1} + A_{t+1}e_{t+1}$, $C_{t+1} = R_{t+1} - A_{t+1}A'_{t+1}Q_{t+1}$, $e_{t+1} = y_{t+1} - f_{t+1}$, $A_{t+1} = R_{t+1}F_{t+1}/Q_{t+1}$.

Step 5: Set $t = t + 1$. If $t = S$, stop ; otherwise, go to **Step 2**.

Therefore, the evaluation of the system status could be done for any time *t* by iteratively running the process from Step 2 to Step 5, given the initial distribution $(\theta_0|I_0) \sim N(m_0, C_0)$. This recursive forecast step of DLM is also shown in Figure 1.

Prediction Model Monitoring

The Bayesian model monitor applied here is based on comparisons of predictions from the standard model M_1 with those from a single alternative model M_2. The latter is constructed sequentially, observation by observation, and is designed as a relatively general and neutral alternative to the specific standard. In essence, the alternative is similar in form to the standard but allows for changes in the values of parameters characterizing the latter. The changes allowed for are designed to be consistent with the types of structural changes expected in the data, yet the alternative model retains neutrality as to the direction and magnitude of such changes.

At time t, the prediction distribution of the observations based on M_1 and M_2 are denoted as $p(Y_t|I_{t-1}, \mathrm{M}_1)$ and $p(Y_t|I_{t-1}, \mathrm{M}_2)$. Given the observation value y_t, the value $p(y_t|I_{t-1}, \mathrm{M}_1)$ is the fundamental measure of predictive ability for the standard model - the model likelihood. Using the likelihood ratio, the predictive ability of M_1 is weighed relative to the alternative model M_2. The likelihood ratio, or Bayes factor is defined as

$$H_t = \frac{p(y_t|I_{t-1}, \mathrm{M}_1)}{p(y_t|I_{t-1}, \mathrm{M}_2)} \tag{3}$$

Small values (< 1) of H_t indicate poor predictive performance of M_1 and, if M_2 is accepted as a sufficiently plausible alternative, M_1 is discredited.

For the sequence of observations $y_1, y_2, \ldots, y_t$ overall model likelihoods may be calculated as

$$L_t = \frac{p(y_1, y_2, \ldots, y_{t-1}, y_t|I_0, \mathrm{M}_1)}{p(y_1, y_2, \ldots, y_{t-1}, y_t|I_0, \mathrm{M}_2} = H_t L_{t-1} \tag{4}$$

Under normal circumstances, the degradation of a structure is a gradual and steady process. However, if there exist certain changes (environmental, loading, etc.) that accelerate the degrading process, the serviceability and remaining lifetime of the structure could be affected. It is crucial to identify the occurrence time of these changes so that proper interventions/maintenances could be undertaken. Since the change of structural performance is relatively small between two adjacent inspection times, the individual difference between Bayes' factors H_t, H_{t+1} will be too small to signal model failure or sequence structure change. The evidence of standard model failure/sequence structure change may take several sampling intervals to become apparent in L_t. Accordingly, to focus on possible local model failure, or to identify a systematic change, it is necessary to consider not only the sequences L_t and H_t, but also the evidence for the standard model for groups of the most recent consecutive observations. This leads to the use of CBF as:

$$H_t(k) = H_t \cdot H_{t-1} \ldots H_{t-k+1} \tag{5}$$

$H_t\,(k)$ assesses the fit of the most recent k observations. To focus on the most likely point of possible change, it is necessary to identify the most discrepant group of recent, consecutive observations. This involves calculating, at time t, the quantity

$$O_t \;=\; \min_{1\le k\le t} H_t\,(k) = H_t\;\min_{1\le k\le t}(1, O_{t-1}\,) \tag{6}$$

For each t, there exists l_t such that $O_t \;= H_t\,(l_t)$. Clearly, $y_t\,, y_{t-1}\;\;, \ldots, y_{t-l_t+1}$ is the most discrepant group of observations of interests. The related run-length parameter$l_t\, = k$ may also be calculated sequentially from

$$l_t \;= \begin{cases} l_{t-1}\; + 1 & if\; O_{t-1}\; < 1 \\ 1 & if O_{t-1}\;\; \ge 1 \end{cases} \tag{7}$$

The run-length l_t provides an indication of the most likely point of onset of change. More details about the Bayesian model monitor can be found in (West and Harrison 1997).

APPLICATION OF DLM IN STRUCTURAL CONDITION ASSESSMENT AND PREDICTION

Corrosion of reinforced concrete bridge beams

Chloride induced corrosion is an important degradation phenomenon affecting the durability of in-service concrete bridges. Rebar corrosion, combined with load variation, will accelerate the degradation process of bridges. Based on Fick's second diffusion law, various chloride diffusion and rebar corrosion models have been proposed considering the effect of several variables including corrosion rate and corrosion initiation time on the time-variant area of the steel reinforcement and flexural strength of an existing reinforced concrete bridge beam. This contribution mainly focuses on the performance of existing bridge beams which are already in the corrosion propagation stage. The corrosion model in (Enright and Frangopol 1998) is employed herein. It is assumed that loss of strength of an element is primarily due to the reduction in the cross sectional area of the steel reinforcement. For a reinforced concrete element with equal bar diameters which have the same corrosion initiation time, the time-variant area of reinforcement steel can be expressed as (Enright and Frangopol 1998):

$$A(t) \;= \begin{cases} \dfrac{n(D(t))^2\pi}{4} & if\; t < D_i\,/(0.0203 \cdot i_{corrr}\,) \\ 0 & if\; t < D_i\,/(0.0203 \cdot i_{corrr}\,) \end{cases} \tag{8}$$

$$D(t){=}D_i\; - 0.0203 \cdot i_{corr}\; \cdot t \tag{9}$$

$$r_{corr}\; = 0.0203 \cdot i_{corr} \tag{10}$$

where n = number of bars, D_i = initial diameter of steel reinforcement, t = elapsed time after corrosion initiation, i_{corr} = corrosion rate parameter and r_{corr} = the corrosion rate of bars.

Take the bridge beam introduced in (Enright and Frangopol 1998) as an example. The reinforced (non-prestressed) T-beam (Figure 2) consists of three 9.1 m simply supported spans, and the bridge consists of five such beams equally spaced 2.6 m apart. The bridge carries an average of 1060 trucks per day. For rectangular non-prestressed members for which the strength of compression steel is neglected, the nominal resistance (flexure) of a concrete beam is given by(AASHTO 1994):

$$R = M_n = A_s f_y (d - \frac{a}{2}) \tag{11}$$

Where $a = A_s f_y / 0.85 f_c' b$, M_n is the nominal resistance, A_s is the area of non-prestressed tension reinforcement, f_y is the specified yield strength of reinforcing bars, d is the distance from the extreme compression fiber to the centroid of non-prestressed tensile reinforcement, f_c' is the specified compressive strength of concrete at 28 days, and b is the width of the compression face of the member. The related variables are listed in Table 1 (Enright and Frangopol 1998).

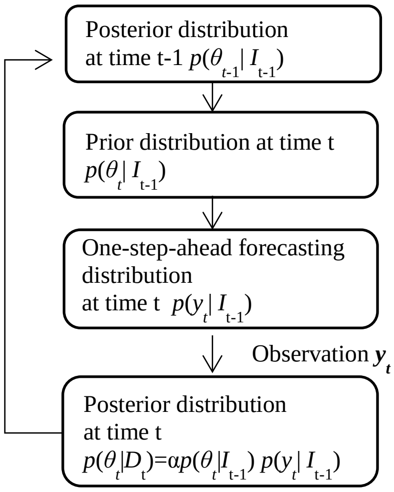

Figure 1. Recursive forecast steps of DLM

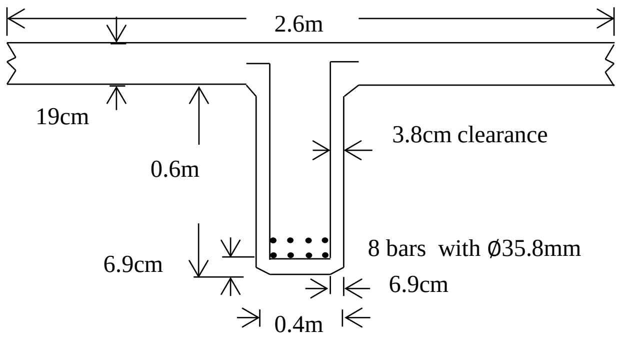

Figure 2. Example reinforced concrete bridge beam

In this paper, we only focus on the phase when the corrosion of the steel bars has already been initiated (at 12 years). The observations of the beam resistance (Enright and Frangopol 1998) were obtained every 2 years after corrosion initiation and from 52 years on, an acceleration of the corrosion rate has incorporated to simulate the change of the sequence structure.

Table 1. Resistance variables for example bridge beam

Variable	Units	Mean	COV	Distribution
f_y	MPa	310.5	0.12	Normal
f_c'	MPa	19	0.18	Normal
D_i	mm	35.8	0.02	Normal
d	cm	68.73	0.03	Normal

Structural performance forecasting with DLM and model monitoring

The degradation of the beam due to corrosion could be characterized with a function g(t) with the form of $g(t) = 1 - \sum k_i t^{\alpha_i}$ where k_i and α_i vary in different models, for example $g(t) = 1 - kt$ in (Enright and Frangopol 1998) and $g(t) = 1 - k_1 t + k_1 t^2$ in

(Enright and Frangopol 1999) . Here we will take $g(t) = 1 - kt$ as standard model M_1 for illustration, and the nominal resistance of the beam could be expressed as:

$$R_t = R_0 g(t) \tag{12}$$

Accordingly we take the second-order polynomial (linear growth) DLM as alternative model M_2. The DLM of the resistance of the beam could be expressed as:

$$y_t = \theta_t + v_t \qquad v_t \sim N(0, V_t) \tag{13}$$

$$\theta_t = G_t \cdot \theta_{t-1} + w_t \quad w_t \sim N(0, W_t)$$

Where $\theta_t=[\mu_t, \beta_t]^T$, μ_t is the resistance level (‰ of R_0) with a variation rate of β_t (‰ of R_0) at time t. In a linear growth DLM, G_t =[1,1;1,0], a 2 × 2 matrix. It could be noted that M_1 is a static model while M_2 a dynamic one with the unknown parameters estimated and updated every time a new observation is available. M_2 is expected to have a better prediction since the parameters could be adjusted to fit the structure changes of the sequence. Consider the following prior information: the initial resistance level 1000‰R_0 with COV of 0.096 , the degradation rate of 2.152 ‰ R_0/year with COV of 0.14, which leads to $(\theta_0|I_0) \sim N\left(\begin{matrix} 1000 \\ -5.304 \end{matrix}, \begin{matrix} 9159 & 69.34 \\ 69.34 & 0.525 \end{matrix}\right)$. Assume $\delta = 0.8$ and V_t estimated sequentially, the one-step-ahead prediction of the resistance of the beam can be done with M_1 and M_2. The inferences and parameter adjustments are achieved with a function developed in Matlab.

Figure 3 shows the one-step-ahead predicted resistance of the beam (‰ of R_0) for M_1 and M_2 respectively as well as the 95% confidence intervals for M_2. Clearly, in normal conditions (before year 52), both methods yield fairly good results, the differences between the predicted resistance and the observed ones are small before year 52 for M_1 and M_2. The predicted value tracks closely the observed resistance across this period and all of the observed data fall into the 95% confidence intervals. The values of the resulting Bayes factors, CBFs and run-length are given in Table 2a. It should be noted that if M_1 was discredited, the Bayes factors would be closer to 0, the natural logarithm of CBFs would be negative and l_t would increase continuously. However, it shows in Table 2a that the Bayes factors and natural logarithm of CBFs fluctuate around 1 and 0, respectively and the run length l_t remains below 3, which means the optimal model changes alternatively between M_1 and M_2. These results also indicate a reliable prior information and an unchanged sequence structure/degradation rate till the year 52.

Figure 4 shows the evolution of CBF which can be found in Table 2b as well. From the l_t sequences in Table 2b there is clear evidence of model failure for M_1 at t = 52. Thereafter l_t continually increases indicating poor subsequent predictive performance of the sequence of standard model. The Bayes factor H_t in Table 2b also shows a tendency of change, but it is not convincing to conclude a change in the sequence structure, since it could also be a result of outliers of observation data. However, the evolution of the CBFs (natural logarithm) excludes that possibility. Normally in Bayes model monitoring (West and Harrison 1997), it is sufficient to conclude the alternative model has a better fit than the standard one if the natural logarithm of CBF is lower than -2. As in this case, after the acceleration of corrosion is incorporated and more observation data become available afterwards, the natural logarithm of CBF decreases significantly from the year 52 onwards. It indicates that the sequence structure has changed and the corresponding l_t value indicates the time when the change starts. This is the time when further check of the degradation rate is needed and the model should be adjusted accordingly. If it crosses the threshold value assigned, certain interventions/maintenances is needed.

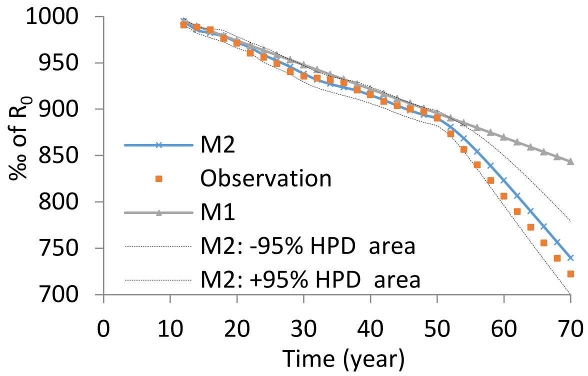

Figure 3. Observation and one-step-ahead prediction with M_1 and M_2

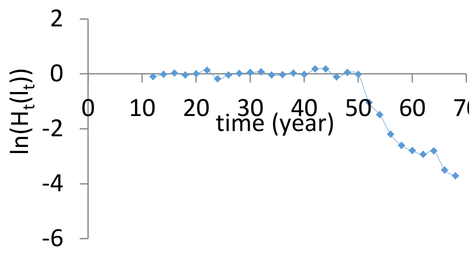

Figure 4. Cumulative Bayes factors of the resistance sequence.

Although, in Figure 3, most of the observation data obtained after year 52 lies within the confidence intervals, M_2 produces wider confidence intervals compared to the period before, reflecting a higher uncertainty in prediction after the change. The reason is that even though the DLM could adjust its parameters based on the newly available observation, it still shares the prior information which affects the predictive ability to some extent.

Table 2. Monitoring the one-step-ahead prediction data

(a)

Time t	34	36	38	40	42	44	46	48
H_t	1,074	0,950	1,018	1,053	0,970	1,224	1,188	0,886
$ln(H_t(l_t))$	0,072	-0,051	-0,033	0,018	-0,030	0,172	0,172	-0,121
l_t	1	1	2	3	1	2	1	1

(b)

Time t	48	50	52	54	56	58	60	62
H_t	0,886	1,129	0,940	0,370	0,644	0,491	0,663	0,830
$ln(H_t(l_t))$	-0,121	0,046	-0,026	-1,056	-1,496	-2,206	-2,618	-2,804
l_t	1	2	1	2	3	4	5	6

CONCLUDING REMARKS

The primary objective of this paper is to develop an effective prediction method using Bayesian dynamic linear model to provide an accurate and reliable structural performance assessment for deteriorating concrete structures. The illustrative example shows that DLM has a better predictive ability compared with a static model. Furthermore, the CBF proves to be feasible to detect abnormalities or systematic changes in structure performance. It could track changes of the sequence structure (evolution trend) even if outlier data at certain time point exist, giving a timely warning when deterioration/structural change occurs and providing information for optimal maintenance strategies.

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